Spatial string tension in gluodynamics at finite temperature in a spherical model approximation

V. K. Petrov*

Bogolyubov Institute for Theoretical Physics,
National Academy of Sciences of Ukraine, Kiev 143, UKRAINE

Spatial string tension is computed in finite temperature gluodynamics on asymmetric lattices in a spherical model approximation. Conditions of scaling behavior are specified. Discrepancies with a standard renormalisation procedure are discussed.

arXiv:hep-lat/9802031v1 21 Feb 1998

I. INTRODUCTION

Recently [1–6] the behavior of the spatial string tension was studied in $(d+1)$-dimensional $(d = 2, 3$) $SU(2)$ and $SU(3)$ gauge theories. The spatial string tension σ at high temperature in $(3+1)$-dimensional $SU(N)$ gauge theory was rigorously proved [3] to be non-vanishing at finite lattice spacing. The spatial string tension, as it was pointed out by [7], was not related to the confining properties of a physical potential in the $(3 + 1)$-dimensional theory. The reason is that under $Z(N)$ - transformation topologically trivial Wilsons loops remain invariant; on the contrary, topologically non-trivial loops such as Polyakov lines transform as $\Omega(\mathbf{x}) \to z\Omega(\mathbf{x})$. Therefore the behavior of topologically trivial Wilsons loops cannot be considered as a confinement criterion [7]. In particular, the expectations of a large space-like Wilsons loop may show area law behavior without static quarks being confined.

Despite the fact that the study of space-like Wilsons loops behavior at finite temperatures does not give straightforward information about critical phenomena in LGT, it helps to understand better non-perturbative effects that manifest themselves in correlation functions for the spatial components of gauge fields.

The remarkable feature of the spatial string tension is that it is scaling of $\frac{\sqrt{\sigma}}{T_c}$ [4] and thus $\frac{\sqrt{\sigma}}{T_c}$ is non-vanishing in the continuum limit. The calculations of an average value of a time- and space-like Wilsons loop at $g^2 \sim g^2_{critical}$ i.e. at fixed cut-off have been performed for a broad temperature interval (T was varied by varying N_τ) for $SU(2)$ [2] and for $SU(3)$ [4] gauge groups. It was shown that the spatial string tension remained temperature independent up to T_c and than was rising rapidly, unlike the temporal one that decreased with temperature above T_c. Similar behavior has been found in lower dimensions and also in $Z(2)$ gauge theory [1,5,6,8].

The main features of high temperature behavior of such observables as the heavy quark potential and spatial

---

*E-mail address: petrov@earthling.net

string tension can be understood in terms of the structure of the effective, three-dimensional theory which was obtained from dimensional reduction at high temperature by means of perturbation theory [9]. The basic suggestion is that at high temperatures temporal dimension becomes arbitrary small and degrees of freedom in that direction are frozen, therefore $\sum_{t=0}^{N_\tau} \sum_{\mathbf{x}} S \approx \frac{1}{a_\tau \cdot T} \sum_{\mathbf{x}} S$ so for three dimensional couplings one can write

$$\beta_{(3)} \equiv N_\tau \beta = \frac{1}{a_\tau T} \frac{2N}{g^2(T)} = \frac{1}{a_\tau} \frac{2N}{g_{(3)}^2}, \qquad (1.1)$$

with

$$g_{(3)}^2 = g^2(T)T. \qquad (1.2)$$

As it was established in [4], the Higgs part of such an effective theory does not contribute substantially in the spatial string tension, leading to the simple relation between σ_3 and σ. One of the remarkable results of [4] can be given as

$$\sqrt{\sigma_3} \approx cN \cdot g_{(3)}^2; \qquad c \sim \frac{1}{5}; \quad N = 2; 3. \qquad (1.3)$$

It means that $\sqrt{\sigma_3}/T_c \approx cN \frac{g_{(3)crit}^2}{T_c} = cNg_{crit}^2$ is in agreement with the MC experiment, which shows that the scaling violation of the ratio $\sqrt{\sigma}/T_c$ is small enough [1].

Though the results obtained in a MC simulation have already answered many crucial questions, we hope that an attempt of an analytic study presented here will be useful for a more detailed understanding of a scaling phenomenon. The present paper is organized as follows. In Sect.2 we discuss the main suggestions, that have been made in the model and compute the average value of a spatial Wilson loop in given approximations. In Sect.3 we discuss the result, obtained for spatial string tension. In Section 4 we make an attempt to give a more comprehensive discussion of the spherical model approximation accuracy and list some standard and nonstandard examples of model applications.

II. MODEL

To compute an average value of the space Wilson loop $W_{R \times L}(\beta_\sigma) = \langle W_{R \times L} \rangle$, we shall use the asymmetrical lattice $\frac{a_\sigma}{a_\tau} = \xi \neq 1$ so for the action one can write

$$S = S_E + S_H;$$

$$S_H = -\beta_\sigma \sum_{t=0}^{N_\tau - 1} \sum_{\vec{x}, t, m \neq n} \frac{1}{N} \mathrm{ReSp}\square_{mn} \qquad (2.1)$$

$$S_E = -\beta_\tau \sum_{t=0}^{N_\tau - 1} \sum_{\mathbf{x}, n} \frac{1}{N} \mathrm{ReSp}\square_{0n}; \qquad (2.2)$$

with

2

$$\beta_\sigma = \beta_{nm} = \frac{2N}{g_\sigma^2 \xi}; \quad \beta_\tau = \beta_{0n} = \frac{2N\xi}{g_\tau^2}. \qquad (2.3)$$

and

$$\Box_{\mu\nu} = U_\mu(x)\, U_\nu(x+\mu)\, U_\mu^\dagger(x+\nu)\, U_\nu^\dagger(x) \qquad (2.4)$$

Lattice spaces a_0 and a_σ can be made arbitrary small at any fixed ξ and no special assumptions are made about g_τ^2 and g_σ^2 except that their values are wholly determined by certain renormalisation group equations at any given a_τ, a_σ, N_τ, and with $N_\sigma^3 \equiv N_1 N_2 N_3$. With decreasing ξ the electric part (S_E) of the action becomes negligibly small in comparison with the magnetic one (S_H), then in $\xi \ll 1$ limit (which in a way is opposite to the Hamiltonian limit: $\xi \gg 1$) it may be ignored. The magnetic part of the action in this case is split into a set of independent time slices. In other words, the temporal degrees of freedom are frozen in the limit $\xi \ll 1$ (which is natural at high temperatures [9]), and we have

$$-S_H = \beta_\sigma \sum_{t=0}^{N_\tau - 1} \sum_{\vec{x},t,m\neq n} \frac{1}{N} \mathrm{ReSp}\Box_{mn}$$

$$\simeq \beta_\sigma \cdot N_\tau \sum_{\vec{x},m\neq n} \frac{1}{N} \mathrm{ReSp}\Box_{mn}. \qquad (2.5)$$

The Wilson loop is placed in one of the slices $t = t_0$ and is not affected by any other slice, therefore this specific part ($t = t_0$) of the action works as an effective action while calculating the average value of Wilson loop. To put it differently

$$Z = \prod_t \mathrm{Sp}_t \left(\exp\left\{ \frac{2}{g_\sigma^2 \xi} \sum_{\mathbf{x},m\neq n} \mathrm{ReSp}\Box_{mn} \right\} \right) = Z(t_0)^{N_\tau},$$

$$(2.6)$$

and $\beta_\sigma = \frac{2N}{g_\sigma^2 \xi}$ can be regarded as the effective coupling. Therefore

$$W_{R\times L}(\beta_\sigma) = \qquad\qquad\qquad\qquad (2.7)$$

$$\frac{1}{Z(t_0)} \mathrm{Sp}_{t_0} \left(W_{R\times L} \exp\left\{ \frac{2}{g_\sigma^2 \xi} \sum_{\vec{x},m\neq n} \mathrm{ReSp}\Box_{mn} \right\} \right).$$

A. Approximation $SU(N) \simeq Z(N)$

It is commonly believed that the $Z(N)$ degrees of freedom are responsible for many important aspects of $SU(N)$ gluodynamics phase structure. The lattice gauge theory in the vicinity of phase transition is widely known to show large degree of universality [7]. The main evidence in favor of universality is given by the Wilson loop behavior, whose functional form does not depend on the choice of a gauge group or on the specifics of ultraviolet

behavior of the model, showing rather simple dependance on space dimension. Universality arguments place the finite - temperature $SU(N)$ - gauge theory in the universality class of globally $Z(N)$ invariant systems with short - range interactions. Hence it is convenient to study such universal infrared behavior in $SU(N) \frown Z(N)$ approximation:

$$U_{x;\mu} \simeq z_{x;\mu} = \exp\left\{\frac{2\pi i q_{x;\mu}}{N}\right\} \;\; ; \;\; q_{x;\mu} = 0, \dots, N-1.$$

(2.8)

The MC experiment as well as model calculations [10] demonstrate that $SU(2)$ and $Z(2)$ spectra show remarkable agreement not only between the pattern of the states, but also between the values of the masses (except for the lowest state) (see also [5] and [6]). Such models with discrete gauge groups are easier to handle. In particular, the method of duality transformations is just the one elaborated well enough for the systems with discrete symmetry ([11] and references there) These models with $Z(N)$ gauge symmetry are also known to provide a transparent realization of 't Hooft algebra of order and disorder operators. Moreover, there is evidence [13] that the effect of the quantum fluctuations near to the $Z(N)$ configurations on the symmetrical lattice leads only to a finite renormalisation of the coupling constant:

$$\beta_{old} \rightarrow \beta_{new} = \beta_{old} - \frac{N^2 - 1}{4}.$$

(2.9)

Since the additional term in (2.9) depends neither on $a_{\sigma,\tau}$ nor on β we may hope that on the asymmetrical lattice the effect of the quantum fluctuations will also lead again to an insignificant change of the coupling constant. We consider it to be still another justification of the chosen approach. Of course, nobody expects full coincidence of the results for $SU(N)$ and $Z(N)$.

One of the main advantages of the $SU(N) \approx Z(N)$ approximation is that one can apply it to $Z(N)$ duality transformations, which relate $Z(N)$ gluodynamics to Ising ($N = 2$) and Potts ($N = 3$) models in 3 - dimensional space .

B. Duality transformations

As it is well known [14], in the case of $Z(2)$ gauge group the Wilson loop $R \times L$ average value placed at $x_1 = 0$ can be calculated on the dual lattice

$$W_{R \times L} = \left\langle \exp\left\{-2\sum_{R \times L} \beta'_n z_{\vec{x}} z_{\vec{x}+n}\right\}\right\rangle ; \;\; z_{\vec{x}} \in Z(2).$$

(2.10)

Summation $\sum_{R \times L}$ is done over all dual Ising spins placed at $x_1 = 0$ inside $R \times L$. In other words, ferromagnetic

links dual to plaquettes of $R \times L$ transform into antiferromagnetic links of the same strength [14]. In the $Z(3)$ gauge theory where the coupling β' is multiplied by $e^{\frac{2\pi}{3}i}$ at links dual to plaquettes placed inside $R \times L$ at $x_1 = 0$ we have

$$W_{R \times L} = \left\langle \exp\left\{ -\left(1 - e^{\frac{2\pi}{3}i}\right) \mathrm{Re} \sum_{R \times L} \beta'_n z_{\vec{x}} z_{\vec{x}+n}^{-1} \right\} \right\rangle.$$

$$(2.11)$$

The duality transformations on the asymmetric lattice ($a_1 \neq a_2 \neq a_3$ and $\beta'_1 \neq \beta'_2 \neq \beta'_3$) relate dual links l_n to plaquettes P_{mk} of the original lattice ($n \neq k \neq m$) [20]. The couplings β'_n on the dual asymmetric lattice are related to the corresponding ones (β_{km}) on the original lattice in the form

$$\beta'_n \approx \begin{cases} e^{-\left(1 - \cos\frac{2\pi}{N}\right)\beta_{km}}; & \beta_{km} >> 1, \\ \frac{1}{N} \ln \frac{1}{\beta_{km}}; & \beta_{km} << 1. \end{cases} \quad (2.12)$$

In the specific case of $a_n = a$, $(\beta'_n = \beta'_\sigma \; ; \; \beta_{km} = \beta_\sigma)$

$$\beta'_\sigma \approx \begin{cases} e^{-\left(1 - \cos\frac{2\pi}{N}\right)\beta_\sigma}; & \beta_\sigma >> 1, \\ \frac{1}{N} \ln \frac{1}{\beta_\sigma}; & \beta_\sigma << 1. \end{cases} \quad (2.13)$$

C. Spherical model approximation

To compute the partition function[1]

$$Z = \sum_{\{z\}} \exp\left\{ \mathrm{Re} \sum_{x,n} \beta'_n z_x z_{x+n}^* + \sum_x \eta_x z_x \right\} \quad (2.14)$$

on the dual lattice we use the well-known spherical model [17] (see e.g. [18], [19] and references therein). The crucial point of this model lies in replacing the precise condition $|z_x|^2 = 1$ by a less burdening "averaged" condition

$$\frac{1}{N_\sigma^3} \sum_x |z_x|^2 = 1. \quad (2.15)$$

or

$$\prod_x \delta(|z_x|^2 - 1) \to \delta\left(\sum_x N_\sigma^3 - |z_x|^2 \right) =$$
$$\int_{c-i\infty}^{c+i\infty} \frac{ds}{2\pi i} \exp\left\{ s\left(\sum_x N_\sigma^3 - |z_x|^2 \right) \right\}, \quad (2.16)$$

where the constant c is chosen to the right of all singularities of the integrand to ensure the correctness of

---

[1]Sources η_x are introduced for convenience.

interchanging the integration over z_x and ds. Now we can change $\sum_{[z_x]} \rightarrow \int_{-\infty}^{+\infty} dz_x$, so the partition function (2.14) may be rewritten in the following form:

$$Z \simeq \int \frac{ds}{2\pi i} e^{sN_\sigma^3} \times \qquad (2.17)$$

$$\int_{-\infty}^{+\infty} \prod_x dz_x \exp\left(-\sum_{x,x'} z_x A_{\vec{x}}^{\vec{x}'} z_{x'} + \sum_x \eta_x z_x \right),$$

where

$$A_{\vec{x}}^{\vec{x}'} = s\delta_{\vec{x}}^{\vec{x}'} - \sum_{n=1}^{3} \beta_n' \delta_{\vec{x}+n}^{\vec{x}'} \equiv$$

$$m^2 \delta_{\vec{x}+n}^{\vec{x}'} - \sum_{n=1}^{3} \beta_n' (\delta_{\vec{x}+n}^{\vec{x}'} - \delta_{\vec{x}}^{\vec{x}'}) \qquad (2.18)$$

with

$$m = \sqrt{s - \sum_{n=1}^{3} \beta_n'} \qquad (2.19)$$

The integration over z_x can be now carried out

$$Z \simeq \int ds \exp\left\{ sN_\sigma^3 - \frac{1}{2} \ln \det A - \eta_x \left(A_{\vec{x}}^{\vec{x}'} \right)^{-1} \eta_{x'} \right\} =$$

$$= \int ds \exp\left\{ N_\sigma^3 \Phi(s) \right\}. \qquad (2.20)$$

To compute the Wilson loop average value, in the partition function $A_{\vec{x}}^{\vec{x}'} = s\delta_{\vec{x}}^{\vec{x}'} - \sum_{n=1}^{3} \beta_n' \delta_{\vec{x}+n}^{\vec{x}'}$ should be changed into $A_{\vec{x}}^{\vec{x}'} + (\Delta_{R \times L})_{\vec{x}}^{\vec{x}'}$, where

$$(\Delta_{R \times L})_{\vec{x}}^{\vec{x}'} = 2\delta_n^1 \delta_{x_1}^0 \delta_{x_1}^{x_1'+1} \theta\left(\frac{R}{2} - x_2 \right) \theta\left(x_2 + \frac{R}{2} \right)$$

$$\cdot \theta\left(x_3 + \frac{L}{2} \right) \theta\left(\frac{L}{2} - x_3 \right). \qquad (2.21)$$

.

After the Fourier transformations

$$\sum_{\vec{x}} \sum_{\vec{x}'} \exp\left(i\vec{\varphi} \cdot \vec{x} - i\vec{\varphi}' \cdot \vec{x}' \right) \cdot A_{\vec{x}}^{\vec{x}'} = A(\varphi_n; \varphi_m') \quad (2.22)$$

we have

$$A(\varphi_n; \varphi_m') = N_\sigma^3 \left[\delta_{\vec{\varphi}'}^{\vec{\varphi}} A(\vec{\varphi}) - \frac{\beta_1'}{N_\sigma^3} \Delta(\varphi_n; \varphi_m') \cos\varphi_1 \right]$$

$$(2.23)$$

with

$$A(\vec{\varphi}) \equiv \sum_{n=1}^{3} \beta_n' \cos\varphi_n, \qquad (2.24)$$

and

6

$$\Delta(\varphi_n; \varphi'_m) = 2 \frac{\sin \frac{(R+1)(\varphi_2 - \varphi'_2)}{2} \sin \frac{(L+1)(\varphi_3 - \varphi'_3)}{2}}{\sin \frac{\varphi_2 - \varphi'_2}{2} \sin \frac{\varphi_3 - \varphi'_3}{2}}. \quad (2.25)$$

The integration over s can be done by the steepest descent method, the saddle point s_0 being defined by the condition $\left[\frac{\partial}{\partial s}\Phi(s)\right]_{s=s_0} = 0$ which can be rewritten as

$$1 = \frac{1}{2} \int_0^{2\pi} \frac{d^3\varphi}{(2\pi)^3} \frac{1}{s_0 - \sum_{n=1}^3 \beta'_n \cos \varphi_n}. \quad (2.26)$$

In the symmetric case the equation (2.26) has a simple solution in the critical point vicinity

$$s_0 \approx 3\beta' + 2\pi^2 \beta' (\beta'_c - \beta')^2 \quad (2.27)$$

with

$$\beta' = \frac{1}{3} \sum_{n=1}^3 \beta'_n$$

or

$$m = \sqrt{s_0 - 3\beta'} \approx \pi\sqrt{2\beta'}(\beta'_c - \beta')\theta(\beta'_c - \beta'). \quad (2.28)$$

The number of sites spanned by the Wilson loop is $1/N_2 N_3$ times smaller than the whole volume, so their contribution doesn't influence the saddle point position.

To clarify the engine of spherical model approach we would note that (2.15)fixing the compactness condition $|z| = 1$ with shown approximation brings(through (2.16)) an additional 'mass' term into action:

$$m_0^2 = -\sum_{n=1}^3 \beta'_n \rightarrow m^2 = s_0(\beta'_n) - \sum_{n=1}^3 \beta'_n. \quad (2.29)$$

The effective mass (2.28) is defined by the saddle point condition (2.26) and plays the role of the screening mass (expressed in lattice units) in the correlation functions.

As it is well known, theories with the local gauge symmetry are described in terms of nonlocal order parameter. Thus the partition function also may have no singularities in the thermodynamic limit, the quantities which determine the nonlocal order parameters may have singularities as a result of increasing their size to infinity. This is strongly suspected to occur in lattice gauge theories for Wilson-loop order parameters, and poses an obstacle to the strong-coupling expansion [21]. Therefore, it seems quite useful to study differences in behavior of a 'tiny' (one-plaquettes) and 'large' (whole $N_2 \times N_3$ plain) Wilson loops. In particular for the 'small' one ($1 << R << N_2$; $1 << L << N_3$) it would be enough to take into account the term $\frac{\Delta(\varphi_n; \varphi'_m)}{N_\sigma^3} \cos \varphi_1$ in the first order

$$\frac{Z_W}{N_\sigma^3} = \frac{Z_0}{N_\sigma^3} W_{R \times L} \approx -\frac{1}{2} \sum_{\vec{\varphi}} \ln\left[s_0 - A(\vec{\varphi})\right] -$$

$$-\frac{RL}{N_\sigma^3} \sum_{\vec{\varphi}} \frac{\beta'_1 \cos \varphi_1}{s_0 - A(\vec{\varphi})}, \quad (2.30)$$

therefore from

$$W_{R \times L} = \exp\left\{ -RLa_2a_3\sigma^{(1)} \right\}, \qquad (2.31)$$

we have for the string tension

$$\sigma^{(1)}a_2a_3 = \beta_1' \int \left(\frac{d\varphi}{2\pi}\right)^3 \frac{\cos\varphi_1}{s_0 - \sum_{\vec{n}} \beta_n' \cos\varphi_n}, \qquad (2.32)$$

This equation can be rewritten as

$$\sigma^{(1)}a_2a_3 = \frac{\beta_1'}{N_\sigma^3} \frac{\partial}{\partial \beta_1'} \ln Z_0. \qquad (2.33)$$

Since the string tension in a certain sense characterizes the average plaquette value , one may anticipate that this quantity is defined (through the duality relations) in the terms of the average value of the link $\mathfrak{L} \equiv \frac{1}{N_\sigma^3} \frac{\partial}{\partial \beta_1'} \ln Z_0$

For the 'large' Wilson loop $(R \approx N_2; L \approx N_3)$

$$\Delta(\varphi_n; \varphi_m') \approx 2RL\delta(\varphi_2 - \varphi_2')\delta(\varphi_3 - \varphi_3'), \qquad (2.34)$$

we have

$$N_\sigma^3 \left[\delta_{\vec{\varphi}'}^{\vec{\varphi}} A(\vec{\varphi}) - \frac{\beta_1'}{N_\sigma^3} \Delta(\varphi_n; \varphi_m') \cos\varphi'_1 \right] =$$

$$N_\sigma^3 \delta_{\varphi_2'}^{\varphi_2} \delta_{\varphi_3'}^{\varphi_3} \left[\delta_{\varphi_1'}^{\varphi_1} A(\vec{\varphi}) - \frac{\beta_1' RL}{N_\sigma^3} 2 \cos\varphi_1' \right], \qquad (2.35)$$

and the free energy $F \equiv -\frac{\ln Z}{N_\sigma^3}$ can be written

$$-F = \int \left(\frac{d\varphi}{2\pi}\right)^3 \ln A(\vec{\varphi}) +$$

$$+ \frac{1}{N_1} \int \frac{d\varphi_2 d\varphi_3}{(2\pi)^2} \ln \left[1 - \frac{\beta_1' RL}{N_2 N_3} \int \frac{d\varphi_1}{2\pi} \frac{\cos\varphi_1}{A(\vec{\varphi})} \right]. \qquad (2.36)$$

The string tension is expressed as

$$\sigma^{(1)}a_2a_3 = \int \frac{d\varphi_2 d\varphi_3}{(2\pi)^2} \ln \left[1 - \frac{\beta_1' RL}{N_2 N_3} \int \frac{d\varphi_1}{2\pi} \frac{\cos\varphi_1}{A(\varphi)} \right]. \qquad (2.37)$$

It is easy to see that for $\frac{RL}{N_2 N_3} \ll 1$ we come back to (2.32)

After the integration over φ_1 and φ_2, one can easily get

$$\sigma^{(1)}a_2a_3 = \zeta(s_0) - \frac{\zeta(s_0 - \beta_1') + \zeta(s_0 + \beta_1')}{2}, \qquad (2.38)$$

where

$$\zeta(s_0) = \int_0^\pi \frac{d\varphi_3}{\pi} \qquad (2.39)$$

$$\ln\left(s_0 - \beta_3' \cos\varphi_3 + \sqrt{(s_0 - \beta_3' \cos\varphi_3)^2 - (\beta_2')^2} \right). \qquad (2.40)$$

In the particular case $(\beta_n' = \beta'; \quad a_n = a)$

$$\sigma^{(1)}a^2 \equiv \sigma a^2 = \zeta(s_0) - \frac{1}{2}\zeta(s_0 - \beta') - \frac{1}{2}\zeta(s_0 + \beta'),$$

$$(2.41)$$

and we obtain

$$\sigma a^2 \approx \begin{cases} c_0 + O(\beta' - \beta_c')^2; & s_0 \gtrsim 3\beta_c' \approx \frac{3}{2}, \\ \frac{1}{2}(\beta')^2 + \frac{3(\beta')^4}{8}; & \frac{\beta'}{s_0} << 1; \end{cases} \quad (2.42)$$

where the constant c_0 insignificantly differs for the 'small' $\left(c_0 \approx \frac{1}{3}\right)$ and for the 'large' $\left(c_0 \approx \frac{1}{4}\right)$ loops.

In the more general case $a_2 = a_3 \neq a_1$

$$\sigma^{(1)} \approx \sigma \cdot \left(\frac{\beta_1'}{\beta'}\right)^2 \left(1 + O(\beta_n')^2\right). \quad (2.43)$$

It is easy see that in order to obtain the corresponding expression for $Z(3)$ the gauge group in given approximation one should change only coupling constant in (2.43) given by (2.12).

As one may anticipate for 'extremely small' loops $(R \sim 1; L \sim 1)$ e.g. single plaquette, approximate expressions for 'small' loops $(2.31)$and $(\mathbf{??})$ become exact.

In a spherical model approximation 'Creutz ratios' are the same for 'large' and 'small' Wilson loops

$$-\ln \frac{W(R,L)\,W(R-1,L-1)}{W(R,L-1)\,W(R-1,L)} =$$

$$= \sigma^{(1)}a_2 a_3 = \beta_1' \int \left(\frac{d\varphi}{2\pi}\right)^3 \frac{\cos \varphi_1}{A(\vec{\varphi})}. \quad (2.44)$$

It should be noted, however, that the difference between 'large' and 'small' Wilson loops has nothing to do with finite size effects, it will survive even at an infinite lattice and reflect alternity between finite Wilson loops and infinite ones. At least in given approximation, such difference is not so dramatic, as it was forewarned in [21].

Comparison with the MC experiment show that spherical model predictions *qualitively* agree with it . Nonanalitical, but quite smooth behavior is demonstrated near the critical point $\sigma(\beta')$. In the critical region $\beta' \sim \beta_c'$ the value $\sigma(\beta') \approx \sigma^c$. In the deep deconfinement region $\beta' << \beta_c'$ (where saddle point steadily moved $s_0 \to 1$) $\sigma(\beta')$ decrease with $g^2 \to 0$ smoothly, but too fast if compared with the MC experiment.

D. Spatial string tension

Here we consider the case of $a_n = a; a_0 = a_\tau = a/\xi$ and put

$$\beta = \frac{2}{g^2}\frac{1}{\xi}; \quad \beta_\tau = \frac{2}{g^2}\xi = \beta\xi^2, \quad (2.45)$$

therefore the expression (2.43) for string tension in spherical model approximation can be rewritten as

$$\sqrt{\sigma} \approx \sqrt{\frac{1}{2}} \cdot \frac{\beta'}{a} \qquad (2.46)$$

which in strong coupling region with (2.12)or (2.13) leads to

$$\sqrt{\sigma} \approx const \cdot \frac{\ln \beta}{a}, \qquad (2.47)$$

therefore $\sqrt{\sigma}$ will scale if we demand $\beta \simeq \exp\left(a \times const\right)$

In weak coupling region the expression (2.46) with (2.12) gives

$$\sqrt{\sigma} \approx \begin{cases} \frac{a^{-1}}{\sqrt{2}} e^{-2\beta}; & Z\left(2\right), \\ \frac{a^{-1}}{\sqrt{2}} e^{-\frac{3}{2}\beta}; & Z\left(3\right) \end{cases} \qquad (2.48)$$

or

$$\sqrt{\sigma} \approx \frac{a^{-1}}{\sqrt{2}} \exp\left\{-\beta\left(1 - \cos\frac{2\pi}{N}\right)\right\} \qquad (2.49)$$

and taking into account

$$g_\sigma^{-2} \approx b_0 \ln \frac{1}{a_\tau \Lambda_L} + ... \qquad (2.50)$$

obtained in [15] and [16] we finally get

$$\sqrt{\sigma} \backsim a_\tau^\epsilon \qquad (2.51)$$

with

$$\epsilon \equiv \frac{2N}{\xi} \cdot \left(1 - \cos\frac{2\pi}{N}\right) b_0 - 1. \qquad (2.52)$$

For $\xi = 1$ the scaling condition $\epsilon = 0$ leads to

$$b_0 = \frac{1/2N}{\left(1 - \cos\frac{2\pi}{N}\right)} \approx \frac{N}{4\pi^2} \approx .025N , \qquad (2.53)$$

which for large enough N agrees with the standard value $b_0 = \frac{11}{12}\frac{N}{4\pi^2} \approx .023N$, so all this looks as if the spatial string tension on asymmetric lattice ($\xi << 1$) acquires 'anomalous dimension' ϵ , which disappears when $\xi \to 1$.

III. DISCUSSION

The application of spherical model in statistical physics has long history since the time it has been introduced to investigate critical phenomena in the ferromagnet [17] and until now (see, e.g. [23], [24]). Although this model is of no direct experimental relevance, it may provide useful insight since many physical quantities of interest can be exactly evaluated with its help. In this context, the spherical model is quite a useful tool in providing explicit verification of general concepts in critical phenomena, see [25] [26] [27] [28].

Recently [23] it was successfully used for studying the transitions between a paramagnetic, a ferromagnetic and an ordered incommensurate phase (Lifshitz point).

Spherical model approximation helped to find the exact scaling function of a system with strongly anisotropic scaling. Models of this kind were investigated extensively (see recent review in [29]).

As it has been established by Stanley [30], there is precise correspondence between the spherical model and the Heisenberg model. Indeed, consider a d - dimensional lattice of N classical spin

$$Z_N^{(\nu)} = \int_{-\infty}^{+\infty} \prod_l \delta\left(\nu - \left|U_l^{(\nu)}\right|^2\right) \times$$

$$\exp\left(-\sum_{l,l'} J_{ll'} U_l^{(\nu)} U_{l'}^{(\nu)}\right) \prod_l dU_l^{(\nu)}. \qquad (3.1)$$

where $U_l^{(\nu)}$ is a ν - dimensional vector of length $\nu^{\frac{1}{2}}$, and $Z_N^{(\nu)}$ is corresponding partition function. In the limit $N, \nu \to \infty$ the free energy of this classical Heisenberg model is identical to that of the spherical model [30]. Kac and Thompson clarified the situation by proving rigorously that this model gives a surprisingly good result for any fixed temperature above and below critical point and is independent of the ordering of the limits $N \to \infty$ and $\nu \to \infty$.

In [24] it was shown that the spherical model is associated with the matrix model [34], which has the same diagrammatic expansion and saddle points in the planar approximation XY model. The analogy with matrix models is interesting because it could provide some useful technology [34] for solving the XY model. In two recent papers [31], [32], Parisi et al. introduced and analyzed the spherical and XY spin models with frustration to test the conjecture that the frustrated deterministic systems at low temperature behave similar to some suitably chosen spin-glass models with quenched disorder [33]

Spherical model predicts reasonable values for critical exponents [18] Moreover, a 'basic' set of exponent relations is also satisfied by spherical model for $d < 5$.

An advantage of the spherical model is that it satisfactorily describes fluctuations and therefore is suitable for the computation of correlation functions either for fixed lattice volume or for infinite one. As it was pointed out by [35] on lattices with infinite volume the perturbation theory (which is in fact a saddle point expansion around an ordered state) gives ambiguous results, and certainly fails for some border conditions.

IV. CONCLUSION

Spatial string tension for 'small' and 'large' loops, as one can expect, shows only marginal difference, which in Creutz ratio can not be discerned at all. This difference, however, is not a lattice artefact and, in principle, can be detected in MC experiment. Spatial string tension power dependence on lattice spacing, demanded

at (2.51), formally does not contradict to condition obtained by standard renormalisation procedure, however, the explicit form of the dependence $\sqrt{\sigma}$ on ξ at (2.51) disagrees with it.

[1] G. S. Bali, K. Schilling, J. Fingberg, U. M. Heller and F. Karsch, e-Print Archive: hep-lat/9308003, *'Computation of the Spatial String Tension in High Temperature SU (2) Gauge Theory'*.

[2] G. S. Bali, J. Fingberg, U. M. Heller, F. Karsch and K. Schilling, Phys. Rev. Lett. **71** (1993) 3059.

[3] C. Borgs, Nucl. Phys. **B261** (1985) 455, *' Area Law for Spatial Wilson Loop in High Temperature Lattice Gauge Theory'*.

[4] F. Karsch, E. Laermann and M. Lütgemeier, Phys. Lett. **B346** (1994) 94 and e-Print Archive: hep-lat/9411020, *'Three-Dimensional SU (3) gauge theory and the Spatial String Tension of the $(3 + 1)$-Dimensional Finite Temperature SU (3) Gauge Theory'*.

[5] M. Caselle, R. Fiore, F. Gliozzi, P. Guaita and S. Vinti, Nucl. Phys. **B422** (1994) 397.

[6] M. Teper, Phys. Lett. **B311**(1993) 223.

[7] B. Svetitsky and L. G. Yaffe, Nucl. Phys. **B 210** [FS6] (1982) 423. *Gauge Theory'*.

[8] L. Kärkkäinen, P. Lacock, D. E. Miller, B. Peterson and T. Reisz, Phys. Lett. **B312** (1993) 173.

[9] T. Reisz, Z. Phys. C - Particles and Fields **53** (1992) 169. *'Realisation of dimensional reduction at high temperature'*.

[10] V. Agostini, G. Carlino, M. Caselle and M. Hasenbusch, DFTT 23/96, HUB-EP-96/24, *'The Spectrum of the $2+1$ Dimensional Gauge Ising model'*.

[11] R. Savit, Rev. Mod. Phys. **52** (1980).

[12] L. Alvarez-Gaume. CERN-TH.7036/93.

[13] T. Yoneya. Nucl. Phys. **B144** (1978) 195.

[14] A. Ukawa, P. Windey, A.H. Guth. Phys. Rev. **D21** (1980) 1013.

[15] A. Hasenfratz and P. Hasenfratz. Nucl. Phys. **B193** (1981) 210

[16] J. Shigemitsu and J. B. Kogut. Nucl. Phys. **B190** (1981) 365

[17] T.H. Berlin and M. Kac, Phys. Rev. **86** (1952) 821.

[18] G. S. Joyce, Phase transitions and critical phenomena. V. 2. London, New York: Academic Press, (1972).

[19] R. Baxter, *Exactly Solvable Models in Statistical Mechanics*, Academic (1982), New York.

[20] L. A. Averchenkova, K. V. Petrov, V. K. Petrov, G. M. Zinovjev, *' Asymmetry parameter role in descrip-*

tion of phase structure of lattice gluodynamics at finite temperature, Jad. Phys. **60** (1997)1, L. A. Averchenkova, K. V. Petrov, V. K. Petrov, G. M. Zinovjev, ' *Lattice asymmetry in finite temperature gluodynamics*, Phys. Rev. **D56** , v.11(1997)56.

[21] J. Kowall and H. Neuberger, Phys. Rev. **D 25** (1982) 2162, ' *Spacelike Wilson loops and roughening'*.

[22] J. B. Kogut, Rev. Mod. Phys., **55**, (1983), 800-802.

[23] L. Frachebourg and M. Henkel, e-Print Archive: hep-lat 9212012,' *Exact Correlation Function at the Lifshitz Points of the Spherical Model'*.

[24] A. Cappelli and F. Colomo, e-Print Archive: hep-lat 971007.

[25] M.N. Barber and M.E. Fisher, Ann. of Phys. **77**, 1 (1973).

[26] S. Singh and R.K. Pathria, Phys. Rev. **B31**, 4483 (1985); **B36**, 3769 (1987).

[27] M. Zanetti and G.F. Mazenko, Phys. Rev. **B35**, 5043 (1987); A. Coniglio and M. Zanetti, Europhys. Lett. **10**, 575 (1989); T.J. Newman and A.J. Bray, J. Phys. **A23**, 4491 (1990).

[28] A.M. Khorunzhy, B.A. Khoruzhenko, L.A. Pastur and M.V. Shcherbina in C. Domb and J. Lebowitz (Eds), *Phase Transitions and Critical Phenomena*, Vol. 15, Academic Press (New York 1992), p. 73 Les Houches XLIX, North Holland (Amsterdam 1990), p. 169; P. Christe and M. Henkel, *Introduction to Conformal Invariance and its Applications to Critical Phenomena*, Springer Lecture Notes in Physics.

[29] W. Selke, in C. Domb and J. Lebowitz (Eds), *Phase Transitions and Critical Phenomena*, Vol. 15, Academic Press (New York 1992), p. 1.

[30] H. E. Stanley, Phys. Rev. **176** (1968) 718.

[31] G. Parisi, J. Phys. **A27** (1994) 7555.

[32] E. Marinari, G. Parisi and F. Ritort, J. Phys. **A28** (1995) 4481.

[33] M. Mezard, G. Parisi and M. A. Virasoro, *Spin Glass Theory and Beyond*, World Scientific (1987), Singapore; G. Parisi, Field Theory, *Disorder and Simulations*, World Scientific (1992), Singapore.

[34] P. Di Francesco, P. Ginsparg and J. Zinn-Justin, Phys. Rep. **C254** (1995) 1.

[35] A. Patrascioi and E. Seiler, Proceedings of the ICHEP '96, World Scientific P.Co (1997) p. 1591, '*The problem of asymptotic freedom'*.